Math 131B-2: Homework 4

Due: April 28, 2014

1. Read Apostol Sections 4.8-13, 4.15-17.
2. Do problems 4.19, 4.21, 4.22, 4.25, and 4.33 in Apostol.
3. Do problems 4.30 in Apostol. Give an example for which the inclusion you proved is not an equality.
4. We say that a subset S of a metric space M is *dense* if every open set in M contains a point of S.
   - Prove that if S is dense in M, every point of M is the limit of a sequence of points in S. (This is very close to a question from the sample midterm.)
   - Prove that if $f : (M, d_M) \to (T, d_T)$ and $g : (M, d_M) \to (T, d_T)$ are two continuous functions from M to a metric space (T, d_T), and $f(s) = g(s)$ for all $s \in S$, then $f = g$ on M.
5. Let $f : X \to \mathbb{R}^n$ be a function such that $f(x) = (f_1(x), \cdots, f_n(x))$. Show that f is continuous if and only if each function $f_i : X \to \mathbb{R}$ is continuous.
6. Suppose that within the borders of a certain country (including on the border itself) there are places in the mountains with heights arbitrarily close to 7000 feet above sea level. Does there need to be a place in these mountains that is exactly 7000 feet above sea level?

   Construct an example of your own this general theme.